Measurement of ion recombination correction factors for plane parallel ionization chambers

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Abstract

The work presented focuses on the measurement of Jaffé plots and determination of ion recombination factors for plane parallel ionisation chambers used in pulsed electron beams. The ability to measure the Jaffé plots accurately is useful when commissioning a new chamber and the ion recombination correction factor needs to be known when the chamber is calibrated in a beam quality different from the user beam.

In this study the ion recombination factor is measured and Jaffé plots are produced for three different plane parallel chambers. A Roos chamber type 34001 (PTW Freiburg), a plane parallel NACP chamber type -02 (Scanditronix) and a plane parallel Pitman chamber (631). Experimental results are presented in form of Jaffé plots, as well as the ion recombination correction factor, determined both from Jaffé plots and by means of the two voltage method as recommended in IAEA TRS-398.

The report also presents results dealing with the measurement technique and experimental setup. Drift, leakage and polarity effect are discussed. The methods of statistical analysis are brought up and discussion is made regarding the validity of the statistical analysis.

The measurements were performed on an Elekta linear accelerator at Perth Radiation Oncology, Wembley, Perth WA. Energies between 6 and 15 MeV were used with doses per pulse below 0.02 cGy per pulse.

The project goal was to measure the ion recombination correction factor and produce a Jaffé plot for a plane parallel Pitman chamber that during various other measurements have shown discrepancy from other chambers. In the goal was also included measurement of the ion recombination factor and Jaffé plot for a plane parallel Roos chamber and a plane parallel NACP chamber to allow for a comparison to be made between chambers. The Pitman chamber was found to have a deviation from linearity of the Jaffé plot at lower voltages than the other chambers.
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1 Introduction

When measuring absorbed dose with an ionisation chamber several corrections need to be made to ensure that the measurement of absorbed dose gives the same result regardless of operational conditions such as radiation quality and dose rate. The International Atomic Energy Agency has published a dosimetry protocol to be used for dose determination in external radiation therapy; IAEA TRS-398. This protocol defines all different correction factors needed to achieve a clinically relevant accuracy of the measurements of absorbed dose. One of the correction factors is the ion recombination factor $k_s$, which corrects for incomplete charge collection at an electrode due to ion recombination in the gas cavity.

The ion recombination correction factor defined in IAEA TRS-398 is assumed to be determinable by the two-voltage method. When using this method one has to suppose that the relationship between the reciprocal of the chamber response and the reciprocal of the polarising voltage is linear. In other words, the user is assuming that the chamber response is consistent with the theory of Boag, which approximately yields a straight line in a Jaffe plot produced for doses per pulse less than 0.1 cGy/pulse [1]. The two-voltage method is derived from the Boag theory. However, for certain chambers and certain voltage regions the relation may be described by a more complex non-linear curve. In the clinic this deviation from linearity may not be significant, however it may be of importance in high precision radiation dosimetry practiced at standards laboratories. In the clinic the ion recombination correction factor determined is further assumed, within an acceptable uncertainty, to be independent of dose per pulse and radiation quality, allowing measurements to be made at different locations in the radiation field using the same correction factor.

Several authors have investigated the dependence of $k_s$ on radiation type. Bruggmoser et al. (2007) [2] and Havercroft et al. (1993) [3] report no dependence of $k_s$ on beam energy. Several authors have also measured the dose per pulse dependency of the ion recombination correction factor. Bruggmoser et al. [2] and Burns & McEwen [4] have for example done measurements for doses per pulse in the order of 1 mGy/pulse.

The ion recombination effect will cause a change in the typical charge measured in a radiotherapy context in the order of 0.2%. To be able to measure the ion recombination effect accurately care has to be taken to account for several influences. The knowledge of error propagation when calculating the correction factor from raw data is necessary for minimizing the uncertainty in the determined ion recombination correction factor. The aim of this study was partly to produce Jaffe plots for three different chambers and determine the error that is associated with the way the ion recombination correction factor is determined and used in the clinic. The study also aims to compare different methods for determination of the ion recombination correction factor and to compare the results from the different chambers.
2 Theory

2.1 Ion Recombination

The method for determining ion recombination correction factors in pulsed beams according to IAEA TRS-398 is based on the Boag Theory. The user only needs to confirm that the reciprocal of the chamber response linearly depends on the reciprocal of the chamber voltage in the voltage region in which the chamber is going to be used and then use Equation 1 which is from IAEA TRS-398 [5]. The chamber response is to be measured for two different voltages; the usual voltage and a lower voltage. The ratio between the two voltages must be of certain integer value. A table provides the coefficients $a_0$, $a_1$ and $a_2$ of the second order polynomial in Equation 1

$$k_s = a_0 + a_1 \frac{M_1}{M_2} + a_2 \left( \frac{M_1}{M_2} \right)^2$$

(1)

where $M_1$ and $M_2$ are the chamber readings at user voltage and at the lower voltage respectively, $k_s$ is the recombination correction factor defined in TRS-398. Derivation of this method, called the standard two-voltage method is given in section 2.1.3.

2.1.1 General and initial recombination

The total effect of ion recombination is a combination of initial and general recombination. However Burns and McEwen (1998) [4] states that; "It is well established that the component of initial recombination also varies approximately linearly in reciprocal current against reciprocal voltage", which is the reason to why the total recombination correction factor $k_s$ can be determined by models assuming linearity between the reciprocal current and the reciprocal voltage as stated in Section 2.1.4. It should also be understood that the two voltage method corrects for the total recombination effect accurately because of this.

2.1.2 Boag’s Theory

Boag’s theory of ion recombination in pulsed beams provides a relation between the correction factor for general recombination $k_{s\text{gen}}$ and chamber voltage, dose per pulse and the distance between the electrodes.

$$k_{s\text{gen}} = \frac{u}{\ln[1 + \frac{e^{p-1}}{p}]}$$

(2)

where $p$ is the free electron fraction and $u$ is given by Equation 3. In Equation 3, $\mu$ is a constant depending on the gas and $d$ is the distance between the electrodes in a plane parallel chamber. The polarising voltage is designated $V$ and $q$ is the amount of charge liberated per volume of air per pulse.

$$u = \frac{\mu d^2 q}{V}$$

(3)
Clinically relevant figures of the different quantities yields a small \( u \). Di Martino et al. [1] showed that Equation 2 can be expanded with a first order Taylor expansion:

\[
k_{\text{gen}} \approx \frac{u}{\ln(1+u)}.
\]  

(4)

### 2.1.3 Derivation of the standard two-voltage method

Suppose that:

\[
k_s = \frac{q_{\text{released}}}{q_{\text{collected}}} = \frac{M_s}{M}
\]  

(5)

In Equation 5, \( M_s \) is the true reading of the chamber and \( M \) is the actual reading of the chamber achieved in practice. Index \( s \) stands for saturation, meaning that the true reading is achieved when all of the charge is collected from the chamber and the saturation current is measured. Using Equation 4 it can be shown that the ratio between the chamber response \( M_1 \) and \( M_2 \) at the two different voltages \( V_1 \) and \( V_2 \) respectively can be written as

\[
\frac{M_1}{M_2} = \frac{\frac{M_1}{M_s}}{\frac{M_2}{M_s}} = \frac{\frac{\ln(1+u_1)}{u_1}}{\frac{\ln(1+u_2)}{u_2}}
\]  

(6)

where \( u \) is defined by Equation 3. Equation 3 also gives the relation \( u_2 = u_1 \frac{V_1}{V_2} \). Substituting \( u_2 \) with this expression gives

\[
\frac{M_1}{M_2} = \frac{V_1}{V_2} \ln(1+u_1) \frac{\ln(1+u_1)}{\ln(1+u_2) u_1}.
\]  

(7)

Solving Equation 7 for \( u_1 \) and inserting numbers for the chamber reading ratio and voltage ratio respectively gives \( u_1 \) to be used in Equation 4 from which \( k_s \) can be obtained. Since Equation 7 is a transcendental equation it can only be solved by numerical (or graphical) methods. Weinhouss and Meli [6] did the work of numerically solving the equation and computing \( k_s \) for different voltage ratios and chamber reading ratios. Their results in the form of the coefficients of Equation 1 are used in IAEA TRS-398.

### 2.1.4 Jaffe diagram

A Jaffe diagram is a plot of the reciprocal of the chamber reading against the reciprocal of the polarising voltage. Approximating Equation 4 for \( k_s < 1.05 \) and consequently \( u < 0.1 \) by expanding the nominator and letting \( u \) tend to zero yields

\[
k_{\text{gen}} \approx \frac{u}{\ln(1+u)} \approx 1 + \frac{u}{2}
\]  

(8)

where \( u \) is proportional to \( \frac{1}{V} \) which together with section 2.1.1 leads to the conclusion that the relation between the reciprocal of the chamber reading and the polarising voltage should be linear for \( k_s < 1.05 \).
3 Materials and Methods

All measurements were done on the Elekta linear accelerator M2 at Perth Radiation Oncology except for the measurements done on the 3rd of December. On the 3rd of December the experiment was carried out on the Elekta Linear accelerator M3 at Perth Radiation Oncology. Three measurements were recorded for each voltage and the average of those three readings are used to produce the diagrams in the report. Three parallel-plate chambers were investigated; one NACP chamber type 02 (Scanditronix) serial number DEA0008703, one Roos chamber (PTW Freiburg) serial number TW34001-1085 and a Pitman chamber serial number 631. The chamber response of the investigated chamber was measured with a Unidos E electrometer (PTW Freiburg) serial number T10009-90316. The reference chamber used simultaneously was connected to a Unidos E electrometer (PTW Freiburg) serial number T10009-90051.

3.1 Accuracy

A number of procedures were implemented to make sure that the quantity was accurately measured. The chamber response was normalised to the response of a reference chamber to adjust for changes in temperature, air pressure and output. The leakage current was measured for each type of experimental setup. The electrometer was allowed to stabilise before measurements were recorded as described in Section 3.1.1. The first voltage of each experiment was repeated at the end to allow correction for drift as described in Section 3.1.2. The readings were corrected for polarity effect at each voltage as described in Section 3.1.3.

3.1.1 Stabilisation of the electrometer

Different methods were used for ensuring that the electrometer had reached equilibrium before measurements were recorded at a new voltage. When the Roos chamber was used with polarising voltages in its linear region, the chamber responses were compared at 10 and 15 minutes after the polarising voltage was changed. Stabilisation was assumed to have occurred when the measurement after 10 minutes of waiting did not deviate more from the three consecutive measurements made after 15 minutes of waiting than the 15 minutes measurements deviated from each other. A more detailed description is that the distances A and B defined in Figure 1 were examined and A was assured to be less than B.
Figure 1: Defining distances A and B used for determining stabilisation time for the electrometer when a chamber was used in its linear region. Distance A was assured to be less than B before stabilisation of the electrometer was assumed to have occurred at a time t minutes after the voltage was set.

When measurements were made on the Roos chamber with polarising voltages above 250 volts (maximum 400 volts), the stabilisation was more carefully observed. Because of drift and a rather big instrument error at these voltages, the main trend of a gradual approach to a stabilisation value was hard to observe for some changes of voltage. However, the chamber response ratio ($M_{Roos}/M_{Ref}$) could be observed to vary around a certain value and was found not to vary more than twice the instrument error. This was except for the two first changes in voltage when the ratio varied more and an extra period of time was spent looking at the stabilisation to see that it was varying around a stable mean value. A stabilisation time between 12 and 30 minutes was applied for the Roos chamber. When doing measurements with the NACP chamber the stabilisation of the electrometer was carefully observed and the ratio was not allowed to vary more than twice the instrument error. Simultaneously the stabilisation graph was observed visually to be sure that the curve was becoming flat. A stabilisation time between 13 and 47 minutes was applied for the NACP chamber. When measurements were made with the Pitman chamber, the ratio was again not allowed to vary more than twice the instrument error. The stabilisation curve was easy to analyse as it showed the trend of approaching a value and flattening out for all changes of voltage. A stabilisation time between 18 and 41 minutes was applied for the Pitman chamber. Examples of stabilisation curves are shown in Figure 2.
Figure 2: Examples of stabilisation curves for the NACP chamber. The chamber response ratio, that is the ratio of the NACP chamber response and the reference chamber response is plotted against time. The time is measured from the change of voltage. In (a) the stabilisation trend is easy to observe, in (b) the stabilisation pattern is not clear.

3.1.2 Correction for drift

The time of the recorded measurement was noted and the first voltage was repeated in the end of the experiment. The drift was found to be very small and a linear time dependence of the drift was assumed. A linear interpolation was made to approximately correct for the drift of the system.

Figure 3 show an example of how the data points is shifted slightly when the drift correction is applied. The ratio of the Pitman chamber response and the reference chamber response is plotted against voltage, in 3(a) no drift correction is applied and in 3(b) drift correction is applied.
The chamber response ratio, that is the ratio of the Pitman chamber response and the reference chamber response, is plotted against voltage. In (a) chamber response ratio is not corrected for drift, in (b) the chamber response ratio is corrected for drift. In both (a) and (b) the error bars are smaller than the data points.

3.1.3 Polarity effect

The polarity correction factor was determined as recommended in IAEA TRS-398 [5]. In other words the differences at negative and positive polarity was assumed to cancel when the readings at the two polarities was averaged to yield the true reading. Examples of the behaviour of the polarity effect can be seen in the Table and Figures in Section 4.1.2.

3.2 Statistics

The error in ratio between the chamber response and the reference chamber response was assumed to be normally distributed around zero. The error of the readings at different voltages was assumed to have the same variance. Consequently the normalised chamber response itself is described by Equation 9. In Equations 9 and 10, $X_{ij}$ is chamber response ratio number $j$ at the $i$th voltage.

$$X_{ij} \sim N(\mu_i, \sigma^2)$$

where $\mu_i$ is the true mean and $\sigma^2$ the true variance of the distribution. To estimate the error in the chamber response, all electrometer readings used to produce one Jaffe plot were used to calculate the sample variance with Equation 10, which is an unbiased consistent estimate of $\sigma$ [7]. Denoting the number of measurements at each voltage $m$ and the number of voltages $k$, the data used to determine the sample variance was a set of $k \times m = n$ measurements. The number of degrees of freedom in the estimated variance is $n - k$ since the true mean is estimated with the sample mean at each voltage. The mean of the measurements at each voltage was determined by Equation 11.
3.2.1 Error Bars

The error in the chamber response was calculated from the sample variance estimated according to Equation 10. The t-distribution was used to obtain the 95% confidence interval of the error of the chamber reading. Because the error in the applied polarising voltage is not known, the error in applied voltage was assumed to be equal to 0.5% (+/-0.25%) based on prior experiences on the clinic.

To estimate the errors in the inverse normalised chamber response and the inverse normalised polarising voltage the error propagation formula in Equation 13 based on work by Arrass was used [8]. Equation 13 was also used when the error was determined for $k_s$ calculated by the means of the two voltage method. In Equation 13 the known errors are the $\sigma_i$’s, $Y$ is the quantity for which the error is being calculated and $f$ is the function which yields $Y$ according to Equation 12

$$Y = f(X_1, X_2, ..., X_i, ...)$$

$$\sigma_Y^2 \approx \sum_i \left( \frac{\partial f}{\partial X_i} \right)^2 \sigma_i^2$$

The y-direction error bars are shown in the diagrams in Section 4 if the error bars are larger than the data points. The x-direction error bars are smaller than the data points in the Jaffe diagrams in Section 4.

3.2.2 Linear Regression

A linear fit was made to the data in the jaffe plots from which $k_s$ was to be determined. The voltage was assumed to be a controlled variable when the linear regression parameters were statistically analysed. The dependent variable, that is the y-axis quantity, was assumed to be normally distributed with the same standard deviation $\sigma$ for all voltages. The regression parameters were calculated in Excel (2003) using the LINEST function. This spreadsheet function determines the best linear fit to the data, with the method of least squares. In other words LINEST fits the straight line in Equation 14 to the data.

$$y = \beta x + \alpha$$

From the deviation of the data points from the linear fit, LINEST also calculates the squared sum of residuals which yields an estimate of the standard deviation of the normal distribution from which the y-axis quantity is assumed to originate. The error bars in the
Jaffe diagrams were calculated according to Section 3.2.1 but the uncertainty in the estimated regression parameters was estimated from the standard deviation received from LINEST. Lindgren & Barry [7] derived the variance of the regression parameters $\alpha$ and $\beta$. The formula for determining the sample variance of $\alpha$ from $\sigma$ is shown in Equation 15.

$$\text{var}\alpha = \sigma^2 \left( \frac{1}{n} + \frac{(\bar{x})^2}{s_x^2} \right)$$

(15)

To estimate the 95% confidence interval for $\alpha$ the $t$-distribution was used [9]. Both $\sigma$ and $\bar{x}$ are estimated from the sample leaving $n-2$ degrees of freedom. The ion recombination factor was determined from $\alpha$ and the error in $\alpha$ could be propagated to the error in $k_s$ by using Equation 13.

### 3.3 Method for estimating the error associated with using a linear model in non-linear Jaffe plots

To get a measure of how a non-linear Jaffe plot affects the determined dose when using the chamber at voltage different from the calibration voltage but assuming the calibration beam quality is the same as the user quality the ratio $R$ is defined below in Equation 16.

When using the chamber at a different voltage than the calibration voltage one must do the ion recombination correction. The ion recombination correction factor, $k_s$, has to be determined for both the calibration voltage and for the user voltage before any absolute dose can be obtained from measurements done at the user voltage. If the method for determining $k_s$ is good enough for clinical use then the dose determined at the user voltage should be sufficiently close to the dose measured at the calibration voltage. This means that correcting the electrometer reading at the user voltage with $k_s$ for the user voltage and correcting the electrometer reading at the calibration voltage with $k_s$ for the calibration voltage and then comparing the two corrected readings, their ratio should be close to unity, assuming that the electrometer readings also are corrected for temperature, pressure and output variations. The ratio is denoted $R$ and is defined in Equation 16.

Using the ratio of the plane parallel chamber response and the reference chamber reading as a relative measure of the chamber response gives a relative chamber response that is already corrected for temperature, pressure and output variations. This relative quantity can be used for calculating $R$:

$$R = \frac{D_u}{D_{cal}} = \frac{k_{su} \cdot N_u}{k_{s cal} \cdot N_{cal}}$$

(16)

where $D_u$ is the dose calculated from measurement done at user voltage and $D_{cal}$ is the dose calculated from measurement done at the calibration voltage. The quantity $N$ is the ratio of the response of the chamber being investigated and the response of the reference chamber, the same indexes apply as for the dose $D$. 

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4 Results and Discussion

4.1 Accuracy

4.1.1 Drift

Very small corrections for drift were applied in most cases. Very small effects, or none at all, were seen on the ability to correct for ion recombination (Section 4.3.2) when data corrected for drift was compared to data not corrected for drift. In one case the drift was larger than the others, this was when the reference chamber IC10/2426 was positioned in water. The magnitude of the drift was found to be approximately 2% during the experiment as shown in Figure 4.

![Graph showing chamber response over time.](image)

Figure 4: The response of the Reference chamber IC10 / 2426 measured continuously during the experiment on the 10th of December, the duration of the experiment was 6.5 hours. The reference chamber IC10 / 2426 positioned in water. The chamber was irradiated with 60 MU 6 MeV electrons at the Electa linac M2, PRO Wembley. SSD equal to 100 cm. Electrometer zeroed after each change in voltage. Time measured from the moment of changing voltage to +400 V over the NACP chamber the first time.

In an attempt to quantify the drift of the Roos chamber, the reference chamber readings were corrected for the drift of the reference chamber (which of course invalidates the pressure and temperature correction). Doing this, the system drift was brought back to very low values indicating that the Roos chamber does not have a significant drift. Also it was found that the IC10/2426 did not drift when positioned in air, confirming that drift correction
is not necessary when the Roos chamber is used in combination with the IC10 positioned in air.

The IC10 positioned in air was however only a useful experimental setup when the temperature in the room changed slowly and by small amounts. When measurements were done in M3’s bunker with no door and a powerful air conditioner the reference chamber response varied as shown in Figure 5

![Figure 5: The response of the Reference chamber IC10 / 2426 measured continuously during the experiment on the 12th of November. The reference chamber IC10 / 2426 positioned in air. The chamber was irradiated with 100 MU of 6 MeV electrons at the Elekta linac M3, PRO Wembley. The SSD was equal to 100 cm. The electrometer was zeroed once before first recorded measurement. Time measured from the moment of changing voltage to +400 V over the NACP chamber the first time.](Image)

It may be possible that the reference chamber is sensitive to the change in air temperature in the bunker and shows the pattern in Figure 5, which can be interpreted as an air conditioner being turned on and off according to a programmed schedule, or thermostat.

The fact that the temperature correction is not made quite accurately when the two chambers are in different surroundings is part of the explanation to the bad behaviour of the polarity effect and deviation from the linear fit of the data points in the Jaffe plots based on data measured with the IC10 positioned in air (that is experiments performed on the 1st, 8th and 22nd of October and the 24th of September).
4.1.2 Polarity effect

After having analysed and corrected for the drift the differences in chamber response for different polarity can be analysed.

Observing the polarity effect is a way of checking the accuracy of the measurement. In Figure 6 the chamber response ratio is plotted against the polarising voltage applied on the Roos chamber. The Jaffe plot in Figure 12 is based on the data in Figure 6.

Figure 6: The ratio between Roos chamber response and reference chamber response plotted against voltage for Roos chamber TW34001-1085. The Roos chamber was positioned at 1 cm effective depth in water and the reference chamber (IC10/2426) with its upper surface leveled with the top surface of the Roos chamber. SSD was set to 100 cm and a 10 cm × 10 cm field was used. The chambers were irradiated with 60 MU of 6 MeV electrons. Measurements done on the 10th of December. Error bars calculated according to section 3.2.1 are the size of the data points. Electrometer zeroed after each change of voltage. The data have been corrected for drift.

The chamber response ratio in Figure 6 is showing a polarity effect that yields higher values for negative polarity than for positive polarity except for 200 and 250 Volts. At those two voltages the data points for different polarity overlap and at 250 Volts the data point for positive polarity even show a probability of being higher than for negative polarity. The polarity effect should not be showing this behaviour if the measurement was accurate. Figure 6 thus suggests that the measurement for 200 and 250 Volts is not accurate with the 250 Volts measurement being the least accurate. This confirms that the data point at 250 Volts should not be trusted when the linear region is evaluated in Figure 12, as is noted in section 4.3.
A similar not well-behaved polarity effect was observed for the measurement on the 8th of October 2008 at depth 4 cm. An explanation might be instability in energy spectrum causing the PDD curve to shift. The bad measurement is not reflected in the Jaffe plot in Figure 19.

In Table 1 the ratio between positive polarity chamber response and negative polarity chamber response \((M_+/M_-)\) for the NACP chamber is shown.

<table>
<thead>
<tr>
<th>Date</th>
<th>50 V (M_+/M_-)</th>
<th>400 V (M_+/M_-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008-11-20</td>
<td>0.9946 (0.0006)</td>
<td>1.0000 (0.0006)</td>
</tr>
<tr>
<td>2008-11-26</td>
<td>0.9968 (0.0003)</td>
<td>1.0008 (0.0003)</td>
</tr>
<tr>
<td>2008-12-03</td>
<td>0.9974 (0.0011)</td>
<td>0.9999 (0.0011)</td>
</tr>
<tr>
<td>2008-12-03 (repeated)</td>
<td>0.9964 (0.0011)</td>
<td>0.9998 (0.0011)</td>
</tr>
</tbody>
</table>

The ratios from experiment performed on the 20th and 26th of November in Table 1 is based on the data shown in Figures 7 and 8. It should be noted that the experiment on 2008-11-26 was performed with the polarising voltage changed in the following order: +400, -400, +350, -350, +50, -50 volts then the first pair of voltages was repeated (+400 and -400 volts) compared to the voltage order of experiment performed 2008-11-20 which was: +400, +350, +50, -50, -350, -400 and +400 volts. During the experiment on 2008-12-03 the chamber response for one magnitude of voltage was measured for the two polarities before changing magnitude as well. The difference in the polarity effect shown in Table 1 may be explained by the drift correction being slightly wrong and having different impact on the result depending on voltage order.

In Figure 7 and 8 are the ratio between the NACP chamber response and the reference chamber response plotted against voltage showing the combined effect of ion recombination and polarity effect for the measurements done on the 20th and the 26th of November respectively.
Figure 7: Chamber response for NACP chamber DEA0008703 plotted against applied polarising voltage. The chamber was positioned in a solid water phantom with reference chamber Farmer 1677 PTW 30013 in build-up cap attached to the phantom surface. The chambers were irradiated with 50 MU of 6 MeV electrons at the Elekta linac M2, PRO Wembley. SSD equal to 100 cm and the NACP chamber positioned at 0.56 cm effective depth. Measurements done on the 20th of November. Error bars calculated according to section 3.2.1 are the size of the datapoints. Electrometer zeroed after each change of voltage.
Figure 8: Chamber response for NACP chamber DEA0008703 plotted against applied polarising voltage. The chamber was positioned in a solid water phantom with reference chamber Farmer 1677 PTW 30013 in build-up cap attached to the phantom surface. The chambers were irradiated with 50 MU of 6 MeV electrons at the Elekta linac M2, PRO Wembley. SSD equal to 100 cm and the NACP chamber positioned at 0.56 cm effective depth. Measurements done on 26th of November. Error bars calculated according to section 3.2.1 are smaller than datapoints. Electrometer zeroed after each change of voltage.

Figure 8 resembles the graph in Figure 7 in that the chamber response for negative voltage is larger than the chamber response for positive voltage at low voltages but the situation is balanced at higher voltages. In Figure 8 the two curves even cross each other. The data in Figure 8 is recorded with an order of the voltages so that not much time has gone by between negative and positive polarity of a certain voltage. Therefore it seems unlikely that this crossing is caused by, for instance, a bad drift correction, instead it is likely that this crossing occurs because of a real change in the sensitivity of the chamber. In other words it is likely that the collecting electrode is so thin that the main cause of polarity effect, namely capturing of primary electrons, is being overbalanced at high voltages by some other effect depending on the magnitude of the polarising voltage. The difference between Figure 8 and Figure 7 may be caused by a bad drift correction in Figure 7.
4.1.3 Leakage

The leakage current was measured for the NACP chamber in water on the 3rd of November and for the NACP chamber in Solid Water phantom on the 20th of November. The leakage current for the Pitman chamber (in perspex phantom) was measured on the 17th of December. The leakage current for the Roos chamber was measured on the 10th of December. The leakage current was in all cases measured several times during the experiment, the leakage current was measured for a new voltage after the electrometer was zeroed. The leakage current for the Reference chamber (different types used at different dates, Table 2 states which reference chamber that was used during the specific experiments) was measured simultaneously. The leakage current was measured over 1 minute or longer.

Table 2: The magnitude of the maximum leakage current for the investigated chamber \( (L) \) and for the reference chamber \( (L_{\text{ref}}) \) measured over 1 minute or longer at some point during the experiment. The electrometers were zeroed before measurement of leakage current for each new voltage. The date of the experiment is given in the table as well as the chamber type being investigated (Type invest) and reference chamber type (Type ref).

<table>
<thead>
<tr>
<th>Date (Type invest/Type ref)</th>
<th>( L ) (nC/min)</th>
<th>( L_{\text{ref}} ) (nC/min)</th>
<th>environment</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008-11-26 (NACP/Farmer)</td>
<td>&lt;0.0017</td>
<td>no signal</td>
<td>solid water</td>
</tr>
<tr>
<td>2008-12-10 (Roos/IC 10)</td>
<td>&lt;0.001</td>
<td>no signal</td>
<td>water</td>
</tr>
<tr>
<td>2008-12-17 (Pitman/Farmer)</td>
<td>&lt;0.001</td>
<td>&lt;0.002</td>
<td>solid water</td>
</tr>
</tbody>
</table>

The leakage current in all cases in Table 2 is not significant when comparing to the measured signal response. The leakage current is not corrected for at any time. This is seen by multiplying the leakage current with the duration of the real signal and compare the error to instrument error. For the Farmer chamber the signal collection time was less than 12 s in all cases, yielding a leakage charge adding to or subtracting from the real signal from the Farmer chamber of less than 0.0004 nC. The instrument error is 0.001 nC for the magnitude of the signal from the Farmer thus the leakage does not need to be corrected for. For the IC 10 reference chamber the leakage current was not detectable when measured over one minute. For the IC 10 reference chamber the signal collection time was less than 20 seconds and the instrument error equal to 0.001 nC in all cases it was used, thus the leakage does not need to be corrected for in that case either. The signal collection time for the Roos chamber was less than 20 seconds and less than 12 seconds in the two cases it was used, and the instrument error was 0.01 nC and 0.001 nC in the two cases respectively. This yields a leakage charge adding to, or subtracting from, the signal of less than 0.0003 nC, which in both cases are much lower than instrument error and thus no correction for leakage is necessary. The signal collection time for the Pitman was less than 10 seconds and the instrument error was equal to 0.001 nC, the leakage charge of magnitude 0.0002 nC thus does not need to be corrected for. The signal collection time for the NACP chamber was less than 12 seconds and the instrument error was equal to 0.001 nC, the leakage charge of less than 0.0004 nC thus does not need to
be corrected for.

The leakage current was also measured during the experiment carried out on the 3rd of December. The NACP was positioned in water as well as the reference chamber IC 10. During this experiment the leakage current was behaving as shown in Tables 3 to 4. The leakage current was found to be negligible except for a few times when the leakage current was measured to suddenly be high during a few seconds and then go back to close to zero again. The results from the leakage current measurements made for a +400 V polarising voltage over the NACP chamber are shown in Table 3. The results from the leakage current measurements made for a −400 V polarising voltage over the NACP chamber are shown in Table 4.

Table 3: Leakage current measured with the electrometer (Unidos E T10009-90316) connected to the NACP chamber for a polarising voltage over the NACP chamber of +400V. The time in the first column refers to the time after zeroing the electrometer which is done 3 minutes after the voltage is changed.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Leak age charge (nC)</th>
<th>Collection time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.000</td>
<td>30</td>
</tr>
<tr>
<td>17</td>
<td>0.026*</td>
<td>60</td>
</tr>
<tr>
<td>18</td>
<td>0.000</td>
<td>60</td>
</tr>
<tr>
<td>19</td>
<td>0.010</td>
<td>60</td>
</tr>
</tbody>
</table>

*The majority of the signal collected during the first 5 seconds

Table 4: Leakage current measured both with the electrometer (Unidos E T10009-90316) connected to the NACP chamber for a polarising voltage over the NACP chamber of +400 V and with the electrometer (Unidos E T10009-90051) connected to the Reference chamber for a polarising voltage over the reference chamber of +250 V. The time in the first column refers to the time after zeroing the electrometers which is done 2 minutes after the voltage is changed over the NACP chamber.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Leak age charge NACP (nC)</th>
<th>Leak age charge Ref (nC)</th>
<th>Collection time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-0.0001</td>
<td>-0.000</td>
<td>60</td>
</tr>
<tr>
<td>6</td>
<td>-0.031</td>
<td>-0.000</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>-0.017</td>
<td>-0.000</td>
<td>30</td>
</tr>
</tbody>
</table>

The leakage currents in the Tables 4 and 3 are at some points too high to be ignored. With a NACP chamber response of 5.15 nC measured during approx 10 seconds the leakage current consequently can constitute up to 1% of the signal. Water leaking into the NACP chamber
could be one explanation for the behaviour of the NACP chamber.

4.2 Statistical analysis

The deviation of the chamber response ratio from the voltage dependent mean was found to yield a straight line in a normality plot and the t-distribution was used in all cases but one (Pitman 17th of December), in which the instrument error was larger than the 95% confidence interval estimated from the t-distribution. In this case the instrument error was used to produce the interval assumed to contain the true reading. The assumption that the residuals in the Jaffe plots were normally distributed was also confirmed after having plotted the residuals in a normality plot.

The error bars calculated according to Section 3.2.1 do not capture variations in the chamber response that only affects the difference in the chamber responses between different voltages and not the chamber responses within one voltage. In other words a Jaffe plot can show linearity and the error bars will not necessarily be bigger. For the same reason a Jaffe plot with much noise does not necessarily have bigger error bars than a Jaffe plot with less noise, because the noise between voltages are not picked up by the statistical analysis. A result of this is seen in Figure 9 which is a Jaffe plot produced on the data from experiment performed 12th of November 2008. This data is noisy because of a temperature change in the room causing the reference chamber to drift as shown in Figure 5.
Figure 9: Normalised chamber response plotted against polarising voltage. The chambers irradiated with 100 MU of 6 MeV electrons at the Electra linac M3, PRO Wembley. SSD equal to 100 cm and the NACP chamber positioned at 1.06 cm effective depth in solid water. Measurements done on the 12th of November. Reference chamber positioned in air. Error bars calculated according to Section 3.2.1, error bars the size of the data points.

The lack of capturing the error in the difference in the chamber responses between different voltages is also evident when errors associated with $k_s$ determined from the two voltage method is calculated. The values of $k_s$ determined from the two voltage method generally have a smaller error associated with them than the values of $k_s$ determined from Jaffe plots, as can be seen in Tables 13 and 14. This is expected since the errors associated with $k_s$ determined by the two voltage method are estimated from precision and not from accuracy.
4.3 Non-linear Jaffe plots

The three investigated chambers yielded the Jaffe plots in Figures 10 to 11 when varying the polarising voltage up to an absolute magnitude of 400 V. The straight line in the plots is a linear fit to the data points corresponding to the three lowest voltages of 50, 100 and 150 volts.

![Jaffe plot](image)

Figure 10: Jaffe plot for Scanditronix chamber NACP-02 DEA0008703 in combination with reference chamber Farmer 1677 PTW 30013. The NACP chamber was positioned at 0.5 cm depth and the reference chamber was in its build up cap attached to the phantom surface. The chambers were irradiated with 50 MU of 6 MeV electrons at the Elekta linac M2, PRO Wembley with SSD equal to 100 cm. Measurements done on the 20th of November. Error bars calculated according to section 3.2.1, error bars the size of the data points. The straight line in the plot is a linear fit to the data points corresponding to the three lowest voltages.
Figure 11: Jaffe plot for Scanditronix chamber NACP-02 DEA0008703 in combination with reference chamber Farmer 1677 PTW 30013. The NACP chamber was positioned at 0.5 cm depth and the reference chamber was in its build up cap attached to the phantom surface. The chambers were irradiated with 50 MU 6 MeV electrons at the Electa linac M2, PRO Wembley with SSD equal to 100 cm. Measurements done on the 26th of November. Error bars calculated according to section 3.2.1, error bars the size of the data points. The straight line in the plot is a linear fit to the data points corresponding to the three lowest voltages.
Figure 12: Jaffe plot for Roos chamber TW34001 - 1085 PTW Freiburg in combination with reference chamber IC10 / 2426. The Roos chamber was positioned at 1 cm depth in water and the reference chamber was positioned with its top-surface leveled with the top-surface of the Roos chamber. The chambers were irradiated with 60 MU of 6 MeV electrons at the Electra linac M2, PRO Wembley with SSD equal to 100 cm. Measurements done on the 12th of December. Error bars calculated according to section 3.2.1, error bars the size of the data points. The straight line in the plot is a linear fit to the data points corresponding to the three lowest voltages.
Figure 13: Jaffe plot for Pitman plane-parallel chamber in combination with reference chamber Farmer 1677 PTW 30013. The Pitman chamber was positioned at 3 cm depth in solid water and the reference chamber was positioned on top of the Pitman chamber inside a slab of solid water. The chambers were irradiated with 30 MU 15 MeV electrons at the Electra linac M2, PRO Wembley with SSD equal to 100 cm. Measurements done on the 17th of December. Error bars calculated according to section 3.2.1, error bars smaller than data points. The straight line in the plot is a linear fit to the data points corresponding to the three lowest voltages.

All three investigated chambers show non-linearity at higher voltages. The non-linearity is similar for all chambers. The similarity can also be seen in Table 12, in which the numbers are fairly similar for the different chambers. However looking at the data points corresponding to 200 V in the different plots it seems as if the Pitman chamber does deviate from the straight line earlier than the others. The linear region for the Pitman chamber is suggested to be below 150 V and the linear region for the NACP chamber is suggested to be below 200 V. The linear region for the Roos chamber is suggested to be at least up to 200 V, the data point for 250 V in Figure 12 seem to be inconsistent with the continous increase the other data points form, and therefore the linear region is hard to estimate for the Roos chamber. Examples of similar non-linear Roos chamber Jaffe plots are shown by G Bruggmoser et al. (2006) [2].

4.3.1 Ion recombination correction factors

When using the data shown in Figures 10 to 13 for determination of $k_s$ no error analysis was made. The calculated $k_s$ determined by the means of the Jaffe plots and by means of the two voltage method are shown in Tables 5 to 8. In the use of the two voltage method the recomendation from IAEA TRS-398 of keeping the ratio between user voltage and reduced
voltage equal to or larger than 3 is followed. When the Jaffe plot is used to determine the ion recombination factor the Jaffe plot is assumed to consist of the data points measured for the voltages equal to and less than the user voltage.

Table 5: Recombination correction factor $k_s$ determined for the NACP-chamber, using measurements done on 20th of November 2008. The two voltage method (TVM) is the method derived by Weinhaus and Meli [6]. The IAEA recommendation of keeping $V_1/V_2$ larger than or equal to 3 is followed. [5] Values of $k_s$ determined with $V_1/V_2$ smaller than 3 is presented in the table but within paranthesis. When using the whole Jaffe plot (JP) for determination of $k_s$ then the Jaffe plot is assumed to consist of the data points measured for the voltages equal to and less than the user voltage.

<table>
<thead>
<tr>
<th>User voltage</th>
<th>$k_s$</th>
<th>TVM</th>
<th>$V_1/V_2$</th>
<th>$k_s$</th>
<th>JP</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>1.0033</td>
<td>4</td>
<td>1.0020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>N/A</td>
<td>-</td>
<td>1.0024</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>1.0043</td>
<td>3</td>
<td>1.0028</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>1.0053</td>
<td>5</td>
<td>1.0036</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>1.0051</td>
<td>4</td>
<td>1.0048</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1.0066</td>
<td>3</td>
<td>1.0066</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>(1.0097)</td>
<td>2</td>
<td>1.0099</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Recombination correction factor $k_s$ determined for the NACP-chamber, using measurements done on 26th of November 2008. The two voltage method (TVM) is the method derived by Weinhaus and Meli [6]. The IAEA recommendation of keeping $V_1/V_2$ larger than or equal to 3 is followed. [5] Values of $k_s$ determined with $V_1/V_2$ smaller than 3 is presented in the table but within paranthesis. When using the whole Jaffe plot (JP) for determination of $k_s$ then the Jaffe plot is assumed to consist of the data points measured for the voltages equal to and less than the user voltage.

<table>
<thead>
<tr>
<th>User voltage</th>
<th>$k_s$</th>
<th>TVM</th>
<th>$V_1/V_2$</th>
<th>$k_s$</th>
<th>JP</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>1.0037</td>
<td>4</td>
<td>1.0020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>N/A</td>
<td>-</td>
<td>1.0021</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>1.0044</td>
<td>3</td>
<td>1.0031</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>1.0053</td>
<td>5</td>
<td>1.0039</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>1.0053</td>
<td>4</td>
<td>1.0048</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1.0068</td>
<td>3</td>
<td>1.0066</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>(1.0099)</td>
<td>2</td>
<td>1.0099</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7: Recombination correction factor $k_s$ determined for the Roos-chamber, using measurements done on 10th of December 2008. The two voltage method (TVM) is the method derived by Weinhouls and Meli [6]. The IAEA recommendation of keeping $V_1/V_2$ larger than or equal to 3 is followed. [5] Values of $k_s$ determined with $V_1/V_2$ smaller than 3 is presented in the table but within paranthesis. When using the whole Jaffé plot (JP) for determination of $k_s$ then the Jaffé plot is assumed to consist of the data points measured for the voltages equal to and less than the user voltage.

<table>
<thead>
<tr>
<th>User voltage</th>
<th>$k_s$</th>
<th>TVM</th>
<th>$V_1/V_2$</th>
<th>$k_s$</th>
<th>JP</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>1.0025</td>
<td>4</td>
<td>1.0020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>N/A</td>
<td>-</td>
<td>1.0012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>1.0031</td>
<td>3</td>
<td>1.0018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>1.0038</td>
<td>5</td>
<td>1.0019</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>1.0032</td>
<td>4</td>
<td>1.0028</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1.0041</td>
<td>3</td>
<td>1.0039</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>(1.0056)</td>
<td>2</td>
<td>1.0057</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Recombination correction factor $k_s$ determined for the Pitman chamber, using measurements done on 17th of December 2008. The two voltage method (TVM) is the method derived by Weinhouls and Meli [6]. The IAEA recommendation of keeping $V_1/V_2$ larger than or equal to 3 is followed. [5] Values of $k_s$ determined with $V_1/V_2$ smaller than 3 is presented in the table but within paranthesis. When using the whole Jaffé plot (JP) for determination of $k_s$ then the Jaffé plot is assumed to consist of the data points measured for the voltages equal to and less than the user voltage.

<table>
<thead>
<tr>
<th>User voltage</th>
<th>$k_s$</th>
<th>TVM</th>
<th>$V_1/V_2$</th>
<th>$k_s$</th>
<th>JP</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>1.0037</td>
<td>4</td>
<td>1.0020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>N/A</td>
<td>-</td>
<td>1.0017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>1.0047</td>
<td>3</td>
<td>1.0017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>1.0047</td>
<td>5</td>
<td>1.0026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>1.0045</td>
<td>4</td>
<td>1.0031</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1.0051</td>
<td>3</td>
<td>1.0049</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>(1.0071)</td>
<td>2</td>
<td>1.0072</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The same properties of the chamber as can be seen in Figures 10 to 13 can also be seen in the calculated values of $k_s$. In the linear region the two methods starts yielding the same results as expected and in the non-linear region the two voltage method yields higher values of $k_s$ than when the Jaffé plot is used for determination of $k_s$. 
4.3.2 Linear model used for non-linear Jaffe plots

The deviation of the ratio $R$ (defined in Equation 16) from unity for different user voltages is shown in Tables 9 to 11. In Tables 9 to 11 the calibration voltage is assumed to be 200 V and the ion recombination correction factor $k_s$ has been determined both by means of the standard two-voltage method and by means of the whole Jaffe plot. The Jaffe plots are assumed to consist of the data points for polarising voltage equal to and lower than calibration voltage.

Table 9: The accuracy (denoted by $R-1$) of $k_s$ determined for the scanditronix chamber NACP-02 DEA00007803, described by the deviation from unity of the ratio between the absolute dose determined at the calibration voltage, assumed to be 200 V, and the absolute dose determined at the user voltage. $k_s$ determined with the standard two voltage method (TVM) [5] as well as from Jaffe plots (JP) (see section 2.1.4)

<table>
<thead>
<tr>
<th>User Voltage</th>
<th>TVM</th>
<th>TVM</th>
<th>JP</th>
<th>JP</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>+0.25</td>
<td>+0.34</td>
<td>+0.15</td>
<td>+0.18</td>
</tr>
<tr>
<td>350</td>
<td>N/A</td>
<td>N/A</td>
<td>+0.13</td>
<td>+0.13</td>
</tr>
<tr>
<td>300</td>
<td>+0.22</td>
<td>+0.18</td>
<td>+0.10</td>
<td>+0.08</td>
</tr>
<tr>
<td>250</td>
<td>+0.18</td>
<td>+0.13</td>
<td>+0.05</td>
<td>+0.04</td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>150</td>
<td>-0.09</td>
<td>-0.10</td>
<td>-0.05</td>
<td>-0.06</td>
</tr>
<tr>
<td>100</td>
<td>(-0.14)</td>
<td>(-0.18)</td>
<td>-0.09</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

Table 10: The accuracy (denoted by $R-1$) of $k_s$ determined for Roos chamber TW34001 - 1085 PTW Freiburg, described by the deviation from unity of the ratio between the absolute dose determined at the calibration voltage, assumed to be 200 V, and the absolute dose determined at the user voltage. $k_s$ determined with the standard two voltage method (TVM) [5] as well as from Jaffe plots (JP) (see section 2.1.4)

<table>
<thead>
<tr>
<th>User Voltage</th>
<th>TVM</th>
<th>TVM</th>
<th>JP</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>+0.26</td>
<td>+0.26</td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>N/A</td>
<td>+0.13</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>+0.19</td>
<td>+0.10</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>+0.22</td>
<td>+0.07</td>
<td></td>
</tr>
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<td>0</td>
<td></td>
</tr>
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<td>150</td>
<td>-0.07</td>
<td>-0.06</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>(-0.20)</td>
<td>-0.14</td>
<td></td>
</tr>
</tbody>
</table>
Table 11: The accuracy (denoted by $R - 1$) of $k_s$ determined for PITMAN plane-parallel chamber, described by the deviation from unity of the ratio between the absolute dose determined at the calibration voltage, assumed to be 200 V, and the absolute dose determined at the user voltage. $k_s$ determined with the standard two voltage method (TVM) [5] as well as from Jaffe plots (JP) (see section 2.1.4)

<table>
<thead>
<tr>
<th>User Voltage</th>
<th>TVM</th>
<th>JP</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>+0.40</td>
<td>+0.37</td>
</tr>
<tr>
<td>350</td>
<td>N/A</td>
<td>+0.21</td>
</tr>
<tr>
<td>300</td>
<td>+0.32</td>
<td>+0.16</td>
</tr>
<tr>
<td>250</td>
<td>+0.15</td>
<td>+0.08</td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>150</td>
<td>-0.27</td>
<td>-0.16</td>
</tr>
<tr>
<td>100</td>
<td>(-0.42)</td>
<td>(-0.26)</td>
</tr>
</tbody>
</table>

Doing the correction for ion recombination as described above takes the user to a dose closer to the true dose determined at calibration voltage when choosing a user voltage different from the calibration voltage. This can be seen when comparing Tables 9 to 11 with Table 12. The deviation from unity of the quantity $R$ is greater in Table 12 than in Tables 9 to 11.

Table 12: The deviation from unity of the ratio between the dose determined at the user voltage and the dose determined at the calibration voltage, when no correction for ion recombination is made, for the scanditronix chamber NACP-02 DEA0008703, the Roos chamber TW34001 - 1085 PTW Freiburg and the Pitman plane-parallel chamber respectively. The deviation is denoted $R_{unc} - 1$ in the table. The calibration voltage is assumed to be 200 V.

<table>
<thead>
<tr>
<th>User Voltage</th>
<th>NACP 20nov $R_{unc} - 1$ (%)</th>
<th>NACP 26nov $R_{unc} - 1$ (%)</th>
<th>Roos 12dec $R_{unc} - 1$ (%)</th>
<th>Pitman 17dec $R_{unc} - 1$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>+0.43</td>
<td>+0.51</td>
<td>+0.34</td>
<td>+0.48</td>
</tr>
<tr>
<td>350</td>
<td>+0.37</td>
<td>+0.43</td>
<td>+0.30</td>
<td>+0.35</td>
</tr>
<tr>
<td>300</td>
<td>+0.30</td>
<td>+0.27</td>
<td>+0.20</td>
<td>+0.30</td>
</tr>
<tr>
<td>250</td>
<td>+0.16</td>
<td>+0.13</td>
<td>+0.16</td>
<td>+0.13</td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>150</td>
<td>-0.24</td>
<td>-0.25</td>
<td>-0.16</td>
<td>-0.33</td>
</tr>
<tr>
<td>100</td>
<td>-0.59</td>
<td>-0.63</td>
<td>-0.43</td>
<td>-0.67</td>
</tr>
<tr>
<td>50</td>
<td>-1.56</td>
<td>-1.62</td>
<td>-0.99</td>
<td>-1.38</td>
</tr>
</tbody>
</table>

None of the three investigated chambers seem to yield non-linear Jaffe plots causing the user to be more than 0.40 % wrong in calculated absolute dose. Considering the case of using the chambers at 50 V without ion recombination correction with a calibration constant
including ion recombination correction all of the chambers would yield an error in the dose of more than 1%.

4.4 Reproducibility of the measurement

Two identical experiments were carried out at different dates for the NACP chamber and the Roos chamber respectively. Jaffe plots were produced and $k_s$ was determined from both the Jaffe plots and by means of the two voltage method. The result was then compared to see if the result was reproducible. In the comparison the data presented in section 4.3 was used for the NACP chamber, for the Roos chamber two new sets of data was used. The voltage was varied up to a magnitude of 250 Volts over the Roos chamber, see the Jaffe plots in Figures 14 to 15. The ion recombination correction factor $k_s$ was determined at 250 Volts for the Roos chamber and at 200 Volts for the NACP chamber.

Figure 14: Jaffe diagram for PTW Roos TW34001 - 1085. The Roos chamber positioned at 1 cm effective depth in water and the reference chamber IC10 / 2426 positioned in air attached to the electron applicator. Chambers irradiated with 100MU 6 MeV electrons in a 10cm × 10cm field size. SSD was set to 100cm. Measurements done 1st of October. Error bars calculated according to section 3.2.1.
From Figures 14 and 15 the ion recombination correction factor can be determined using the y-axis intercept of the linear fit and by means of the two voltage method, see results in Table 13. In the two voltage method the data points for 250 V and 50 V were used, this is with $V_1/V_2$ equal to 5.

Table 13: The ion recombination correction factor $k_s$ determined for Roos chamber (TW34001-1085) operated at 250 V from the data in Figures 14 and 15 with the error equivalent to the width of the 95% confidence interval. The ion recombination factor was determined both from the Jaffe plots (JP) and by means of the two voltage method (TVM). $V_1/V_2$ equal to 5.

<table>
<thead>
<tr>
<th>Date</th>
<th>$k_s$ (JP)</th>
<th>error $k_s$ (JP)</th>
<th>$k_s$ (TVM)</th>
<th>error $k_s$ (TVM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008-10-01</td>
<td>1.0033</td>
<td>0.0005</td>
<td>1.0040</td>
<td>0.0002</td>
</tr>
<tr>
<td>2008-09-24</td>
<td>1.0028</td>
<td>0.0005</td>
<td>1.0039</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

The results differ depending on which method that is used. This can also be seen by looking at the Figures 14 and 15 in which the linear regression line does visibly separated from the data points corresponding to the voltages used in the two voltage method. The magnitude of the difference seen in the Figures is only approximately 0.03% and cannot be the whole explanation to non-consistency between the two methods. From an experimental point of view the agreement between the two dates is good but a two sample t-test yields a p-value below 0.03%. 

Figure 15: Jaffe diagram for PTW Roos TW34001 - 1085. The Roos chamber positioned at 1 cm effective depth in water and the reference chamber IC10 / 2426 positioned in air attached to the electron applicator. Chambers irradiated with 100MU 6 MeV electrons in a 10cm x 10cm field size. SSD was set to 100cm. Measurements done on the 24th of September. Error bars calculated according to section 3.2.1.
0.001 when comparing results from the two different dates and more accurate measurements are required to get statistical agreement. It should be understood that during the experiments in which the voltage over the Roos chamber is kept below 250 V a different assumption the stabilisation of the electrometer is not observed and a less rigid method for assuring stability is used as described in Section 3.1.1. This is the reason for why the non-linearity showing in Figure 12 cannot be seen in Figures 14 and 15. Doing a more accurate measurement by observing the stabilisation of the electrometer would yield approximately the same error in the data points but the determined $k_s$ would be more similar.

The results from the two identical experiments with the NACP chamber are compared in Table 14. In the two voltage method the data points for 200 V and 50 V were used.

Table 14: The ion recombination correction factor $k_s$ determined for NACP chamber (DEA0008703) operated at 200 V from the data corresponding to polarising voltage 200 V and lower in Figure 11 and 10 with the error equivalent to the width of the 95% confidence interval. The ion recombination factor was determined both from the Jaffe plots (JP) and by means of the two voltage method (TVM). $V_1/V_2$ equal to 4.

<table>
<thead>
<tr>
<th>Date</th>
<th>$k_s$ (JP)</th>
<th>error</th>
<th>$k_s$ (JP)</th>
<th>error</th>
<th>$k_s$ (TVM)</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008-11-26</td>
<td>1.0048</td>
<td>0.0005</td>
<td>1.0051</td>
<td>0.00001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008-11-20</td>
<td>1.0048</td>
<td>0.0005</td>
<td>1.0051</td>
<td>0.0002</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparing the results from different dates in Tables 13 and 14 the results seem to be reproducible. A two sample t-test yields a p-value close to unity. This can be interpreted as that the careful stabilisation observation has paid off. Comparing the ion recombination correction factor determined with different methods the NACP chamber results are satisfying.

4.5 Energy dependence

During the experiment performed on 1st of October 2008 measurements were carried out for different energies. After changing voltage and having waited for the chamber to stabilise the chambers were irradiated with the energies 6, 8, 10, 12 and 15 MeV in that order for each voltage respectively. The results are presented in table 15. The results for 6 Mev in this section is based on the same 2008-10-01 data in section 4.4.
Table 15: The ion recombination correction factor $k_s$ determined from Jaffe diagram, with the error equivalent to the width of the 95% confidence interval, for different energies. Measurements done on the 1st of October 2008.

<table>
<thead>
<tr>
<th>Energy (meV)</th>
<th>$k_s$</th>
<th>error in $k_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.0033</td>
<td>0.0005</td>
</tr>
<tr>
<td>8</td>
<td>1.0033</td>
<td>0.0005</td>
</tr>
<tr>
<td>10</td>
<td>1.0030</td>
<td>0.0005</td>
</tr>
<tr>
<td>12</td>
<td>1.0030</td>
<td>0.0005</td>
</tr>
<tr>
<td>15</td>
<td>1.0020</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The ion recombination correction factors determined for different energy in Table 15 are significantly different from one another. Using a two sample t-test yields p-values smaller than 0.05 which suggests difference in the determined ion recombination correction factors with a 95% confidence level. However this difference was revised to be insignificant due to inaccuracy as noted in Section 4.4. The $k_s$ value at 15 MeV was determined from the data shown in Figure 16. The poor fit to the linear regression line has been reflected in the error and this is possibly a sign for non-accurate measurement. Looking at Figure 16, from which the data in the 15 MeV row in Table 15 is calculated, the poor fit to the linear regression line is showing as something that could be non-linearity. More measurements were done for 15 MeV to see if the non-linearity was reproduceable, see section 4.5.1, measurements were made for 15 MeV on different depths to determine $k_s$ for different energies without changing any settings on the linac.
Figure 16: Jaffe diagram for PTW Roos TW34001 - 1085. The Roos chamber positioned at 1 cm effective depth in water and the reference chamber IC10 / 2426 positioned in air attached to the electron applicator. Chambers irradiated with 100MU 15 MeV electrons in a 10cm × 10cm field size. SSD was set to 100cm. Measurements done on the 24th of September 2008. Error bars calculated according to section 3.2.1 The solid line is a linear fit to all data points.

4.5.1 Varying depth for 15MeV

Varying depth between 1 cm, 2 cm and 4 cm and keeping the beam energy constant at 15 MeV yielded the graphs shown in Figure 17 to 19. The chamber was kept at depths where the PDD was flat to avoid changes in dose per pulse. The dose per pulse was kept nearly constant at 0.016 cGy/pulse which the machine is calibrated to yield at 15 mm depth in water. At 10, 22 and 40 mm depths the PDD normalised to the dose at 15 mm is 98.6%, 101.3% and 93.5% respectively.

The depths were also chosen to make the energy spectrum hitting the chamber approximately correspond to nominal beam energy 10 MeV (22 mm) and 6 MeV (40 mm) respectively. This made it possible to see if the non-linearity in Figure 16 showed at those energies as well when the linac is set to produce electrons with nominal energy 15 MeV. In other words the one parameter that is changed is the energy. The settings on the linac as well as the dose per pulse is kept constant.
Figure 17: Jaffe diagram for PTW Roos TW34001 - 1085. The Roos chamber positioned at 1 cm effective depth in water and the reference chamber IC10 / 2426 positioned in air attached to the electron applicator. Chambers irradiated with 100MU 15 MeV electrons in a 10cm × 10cm field size. SSD was set to 100cm. Measurements done on the 8th of October 2008. Error bars calculated according to section 3.2.1.
Figure 18: Jaffe diagram for PTW Roos TW34001 - 1085. The Roos chamber positioned at 2.2 cm effective depth in water and the reference chamber IC10 / 2426 positioned in air attached to the electron applicator. Chambers irradiated with 100MU 15 MeV electrons in a 10cm × 10cm field size. SSD was set to 100cm. Measurements done on the 8th of October 2008. Error bars calculated according to section 3.2.1.
Figure 19: Jaffe diagram for PTW Roos TW34001 - 1085. The Roos chamber positioned at 4 cm effective depth in water and the reference chamber IC10 / 2426 positioned in air attached to the electron applicator. Chambers irradiated with 100MU 15 MeV electrons in a 10cm × 10cm field size. SSD was set to 100cm. Measurements done on the 1st of October 2008. Error bars calculated according to section 3.2.1.

It seems as if the non-linearity showing in Figure 16 was only showing because of inaccurate measurement. The non-linearity could not be reproduced with any consistency in Figures 17 to 19. The data in Figures 17 to 19 yields the $k_s$ determined from the Jaffe plots shown in Table 16.

Table 16: The $k_s$ determined from Jaffe diagram, with the error equivalent to the 95% confidence interval, for different depths. Measurements done on the 1st of October 2008, electron energy equal to 15 MeV.

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>$k_s$</th>
<th>error in $k_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0033</td>
<td>0.0006</td>
</tr>
<tr>
<td>2.2</td>
<td>1.0030</td>
<td>0.0009</td>
</tr>
<tr>
<td>4</td>
<td>1.0025</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

No statistical agreement can be achieved for the values of $k_s$ determined at different depths with electron energy 15 MeV two sample t-tests yield p-values close to zero.
4.6 Dose Per Pulse Dependence

To be able to investigate the dependency of $k_s$ on dose per pulse measurements were done for the depths 1.4 cm, 2.3 cm and 2.8 cm at a different SSD compared to earlier measurements. This would yield a set of $k_s$ determined at a set of lower dose per pulse, namely a dose per pulse of approximately 50%, 30% and 3% of 0.016 cGy/pulse. At 1.4 and 2.3 cm depths the chamber was irradiated with 200 MU and at 2.8 cm the chamber was irradiated with 700 MU. The dose per pulse at those depths was calculated comparing the roos chamber response to the response recorded at known dose per pulse. The results are shown in Table 17.

Table 17: The $k_s$ determined from Jaffé diagram, with the error equivalent to the 95% confidence interval, for different depths and thus different dose per pulse. Measurements done on the 22nd of October 2008, nominal electron energy equal to 6 MeV.

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>$k_s$</th>
<th>error in $k_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>1.0024</td>
<td>0.0007</td>
</tr>
<tr>
<td>2.3</td>
<td>1.0051</td>
<td>0.005</td>
</tr>
<tr>
<td>2.8</td>
<td>1.0073</td>
<td>0.009</td>
</tr>
</tbody>
</table>

Collecting $k_s$ from all previous measurements as well as from the measurement done at 22nd of October and plotting them against DPP yields the graph in Figure 20.

Figure 20: $k_s$ plotted against dose per pulse. $k_s$ determined from jaffé plots produced from measurements done 1st, 8th and 22nd October 2008. The error bars corresponds to the 95% confidence interval.
Because of lack of statistical agreement when reproducing the measurement no dose per pulse dependence can be obtained from the data in Figure 20.
5 Concluding Remarks

When stabilisation of the electrometer is carefully observed, consistency between the two methods of determination of $k_s$, that is by means of a Jaffé plot or by using the two voltage method, is achieved as shown in the NACP case in Section 4.4. However when the two methods are used on data such as in Figure 14 and 15 the results deviate so much that the use of the sophisticated two voltage method seem not to be entitled, (it seems the approximation of the two voltage method which is also given in IAEA TRS-398 would be good enough). One explanation for the deviation can be that the fit to numerical solutions of Equation 7 done by Weinhouss and Meli does deviate for certain voltage and charge ratios as much as 0.1% from the numerical solution [6]. The error analysis used in the report also yields confidence intervals when this technique of measurement is used that questions the use of the sophisticated two voltage method. The reason for the recommendation of the use of the two voltage method in IAEA TRS-398 would be interesting to examine.

Moreover the statistical analysis used in the report confirms that the ion recombination effect is quantifiable and that the non-linearity of Jaffé-plots can be seen when using this technique for measuring the chamber response dependency on polarising voltage. To better account for variations in the difference between the chamber response at different voltages measurements at more different voltages would be desireable instead of additional degrees of freedom because of multiple readings at few voltages. However it may be good to consider that the measurements to produce Jaffé plots will then be taking more time. The longer the duration of the measurement the more the system is able to drift.

No conclusions on the dose per pulse dependence of the ion recombination correction factor could be determined in this study. The error in $k_s$ at low dose per pulse is too large as shown in Figure 20. This might be due to instability of the energy spectrum produced by the linac, causing the PDD curve to shift slightly. At points on the steep part of the depth dose curve the measurement of dose is more sensitive to any instability in energy. A better way to do the measurements, it seems, would be changing SSD instead of depth.

For measurements in the future it should be understood that the NACP chamber and the IC10 should be used with care when positioned in water. The behaviour of the IC10 seem to be predictable, an explanation to the drift could be that the active volume increases with time due to water diffusion into the encapsulation. The NACP chamber seems to be very unpredictable with a leakage current of large variation when in water. Since no problems with charge storage seem to have occured when the NACP chamber were positioned in solid water it can be advised to do so. However an experimental set up where the reference chamber can be positioned in the same environment is recommended after having seen the results with the reference chamber positioned in air in section 4.1.1.
The chamber comparison in the study suggests that the Pitman chamber is of worst design with a linear Jaffe plot only below 150 V. The NACP chamber and the Roos chamber more comparable in design are surprisingly showing very different ion recombination correction factors. It is worth pointing out though that the measurements in solid water was made after measurements in water. Since the measurements in water suggested leakage of water into the chamber one should perhaps not trust the chamber after that. A more thorough evaluation of the chambers design can perhaps lead to a better understanding in how the polarity effect should show and that way one may understand if the chamber has been affected by the water leakage.

This report can recommend to observe the stabilisation of the electrometer when producing a Jaffe plot. The results from the measurements done with the Roos chamber in its linear region are statistically different. The variation could be avoided by letting the electrometer stabilise, as was done for the other chambers.
Acknowledgement

I would like to thank my supervisor Sean Geoghegan for giving me the opportunity to come to The Royal Perth Hospital in Perth, to do this project and complete my Master’s thesis. I have really appreciated all the assistance Sean Geoghegan has given during the project and I admire his preparedness to supervise me during late nights of measurements. I would also like to thank all the other physicists in the department for interesting comments and discussions. The department has a supporting and hostile group of staff and have made me as a student feel welcome.

I would also like to thank my friend Frida Astrand for being a helpful and supporting travel companion, house mate and fellow student.
References


APPENDIX

Justification of the use of the t-distribution

5.0.1 The t-distribution

If $Z \sim N(0, 1)$, $V \sim \chi^2_\nu$ and $Z$ and $V$ are independent then

$$T = \frac{Z}{\sqrt{V/\nu}}$$

is t-distributed with $\nu$ degrees of freedom.

5.0.2 Cochran’s theorem

Let $X_1...X_n$ be independent normally distributed random variables with mean $\mu$ and variance $\sigma^2$. Then $U_i = \frac{X_i - \bar{X}}{\sigma}$ is standard normally distributed (mean equal to 0 and variance equal to 1). The squared sum of the $U_i$’s can be written as shown in equation A-2

$$\sum_{i=1}^{n} U_i^2 = \sum_{i=1}^{n} \left(\frac{X_i - \bar{X}}{\sigma}\right)^2 + n \left(\frac{\bar{X} - \mu}{\sigma}\right)^2$$  (A-2)

$$\therefore \sum_{i=1}^{n} \frac{(X_i - \mu)^2}{\sigma^2} = \sum_{i=1}^{n} \left(\frac{X_i - \bar{X}}{\sigma}\right)^2 + n \left(\frac{\bar{X} - \mu}{\sigma}\right)^2 = Q_1 + Q_2$$  (A-3)

In equation A-3 it can be seen that $Q_1$ and $Q_2$ are sum of squares of linear combinations of the $U_i$’s. Because the unknown $\mu$ is estimated with $\bar{X}$, one of the $U_i$’s can always be written as a linear combination of the rest of the $U_i$’s, this means that $\text{rank}[Q_1] = n - 1$ and $\text{rank}[Q_2] = 1$. The criteria for Cochran’s theorem that the sum of the ranks should be equal to the number of $U_i$’s is met and Cochran’s theorem now states that $Q_1$ and $Q_2$ are independent and chi-squared distributed with $(n - 1)$ and 1 degrees of freedom respectively.

5.0.3 Subgroups

If the data set is divided into subgroups so that each subgroup is normally distributed with an identical variance but around different means, then Cochran’s theorem and the properties of the chi-squared distribution will still confirm that the t-distribution is describing the behaviour of the error. If the number of subgroups are $k$ and the number of observations within each group is $m$ (number of observations $= m \times k = n$) then equation 10 can be used to obtain an unbiased estimate of $\sigma$. Once $\sigma$ is determined the t-distribution can be used as shown below.

The mean of each subgroup $\bar{X}_k$ can be transformed to the standard normally distributed variable $Z$ according to equation A-4.

$$Z = \frac{\bar{X}_k - \mu_k}{\sigma/\sqrt{m}}$$  (A-4)
Since $\sigma^2$ is unknown and has to be estimated by the sample variance, the variable that has to be analysed is $T$ defined in equation A-5

$$T = \frac{\bar{X}_k - \mu_k}{s/\sqrt{m}} \quad \text{(A-5)}$$

To allow for the use of the t-distribution the variable $T$ has to be shown to be described by the t-distribution and the degrees of freedom has to be found. $T$ can be rewritten as seen in equation A-6

$$T = \frac{\bar{X}_k - \mu_k}{\sigma/\sqrt{m}} \cdot \frac{1}{\sqrt{(n-k)s^2}} \cdot \frac{1}{s} \quad \text{(A-6)}$$

The first factor on the right hand side of Equation A-6 can be recognised as $Z$ and as stated above a variable that is standard normally distributed. Now $(n-k)s^2/\sigma^2$ has to be shown to be described by the chi-squared distribution with $n-k$ degrees of freedom.

One property of the chi-squared distribution is that a sum of $\chi^2$-distributed random variables is again $\chi^2$-distributed now with the number of degrees of freedom equal to the sum of the degrees of freedom of the different summed variables. That is; if

$$Y = \sum_{x=1}^{N} X_x \quad \text{(A-7)}$$

where $N$ is the number of observations, the $X_x$:s are independent and

$$X_x \sim \chi^2(\nu_x) \quad \text{(A-8)}$$

then

$$Y \sim \chi^2(\bar{\nu}) \quad \text{(A-9)}$$

where

$$\bar{\nu} = \sum_{x=1}^{N} \nu_x \quad \text{(A-10)}$$

This very useful property of the chi-squared distribution combined with Cochran's theorem (see Section 5.0.2) confirms that the t-distribution can be used. The inner summation of Equation 10 is chi-squared distributed with $m-1$ degrees of freedom according to the Cochran Theorem. Using the property of the chi-squared distribution (described in Equations A-7 to A-10) it is found that $s^2(n-k)$ is chi-squared distributed with $(m-1) \cdot k$ degrees of freedom.